HEAT- AND MASS-TRANSFER PROBLEM FOR PLANE COUETTE FLOW IN THE SUBLIMATION OF A MOVING PLATE

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The problem of heat and mass transfer in a narrow slit channel with sublimation of a moving plate is solved in an approximate formulation. The influence of the velocity of plate motion on the efficiency of sublimational cooling is investigated.

In designing various cryogenic devices, it is necessary, in many cases, to investigate the heat and mass transfer in a narrow slit channel formed by a moving wall with sublimating material and a motionless parallel wall/heater.

In [1], a one-dimensional problem of this type was solved and experimentally tested. In the present work, the study of the possibility of intensifying the sublimational cooling by moving the sublimating wall along the heated plane is continued.

Consider a steady two-dimensional flow of gas vapor in a narrow slit of a plane channel bounded by two parallel plates (Fig. 1). It is assumed that sublimation of solidified gas occurs from one of the channel walls, moving in its plane at a constant velocity U*, while the other wall is under the action of a uniformly distributed constant heat flux q. Both channel boundaries are regarded as isothermal over the whole length, and the sublimation temperature of the solidified gas T_s is assumed to be constant over time. In this case, the temperature field in the channel will be one-dimensional, depending only on the transverse coordinate y. In addition, in view of the slight temperature drop between the slit walls, consideration is limited to a flow of subliming vapors with constant thermophysical characterisitcs. Taking account of these assumptions, the equations of heat and mass transfer in the quasisteady approximation in the absence of mass forces may be written in the following dimensionless form

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{1}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

$$v\,\mathrm{Pe}\frac{dT}{dy}=\frac{d^2T}{dy^2}. \tag{4}$$

Here $\mathrm{Re} = Hv_s/\nu$ and $\mathrm{Pe} = Hv_s/a$ are, respectively, the Reynolds number and the Peclet number, including the velocity of gas injection from the subliming wall v_s, which is assumed to be known. In reducing Eqs. (1)-(4) to dimensionless form, the velocity components are referred to v_s and the temperature to its value at the sublimation surface. The system in Eqs. (1)-(4) is closed by the condition of adhesion at the motionless and moving channel walls

$$u=0,\ v=0 \text{ when } y=0, \tag{5}$$

$$u=U^*/v_s,\ v=1 \text{ when } y=1 \tag{6}$$

and two conditions for the temperature at the subliming surface

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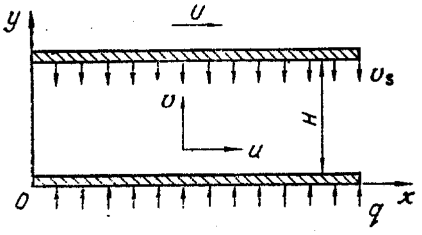

Fig. 1

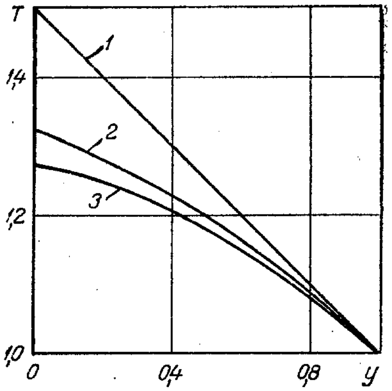

Fig. 2

Fig. 1. Flow in a slit channel with sublimation of a moving wall.

Fig. 2. Temperature distribution over the slit height: 1) absence of sublimation (Pe = 0); 2) Poiseuille flow (Pe = 1, U = 0); 3) Couette flow (Pe = 1, U = −2).

$$T = 1, \quad y = 1, \tag{7}$$

$$dT/dy = -m, \; y = 1. \tag{8}$$

The first of these reflects the constancy of the sublimation temperature and the second the heat-balance condition obtained under the assumption of a small heat flux in the volume of a subliming body [2].

Consider the system in Eqs. (1)-(3) with the boundary conditions in Eqs. (5) and (6). The continuity equation — Eq. (3) — is satisfied identically if the dimensionless function defined as follows is introduced [2]

$$u = -xf'(y), \quad v = f(y). \tag{9}$$

In this notation, the equations and boundary conditions of the mass-transfer problem take the form

$$f''' + (f'^2 - ff'')\,\mathrm{Re} = -\frac{1}{x}\frac{\partial p}{\partial x}\,\mathrm{Re}, \tag{10}$$

$$f'' - ff'\,\mathrm{Re} = \frac{\partial p}{\partial y}\,\mathrm{Re}, \tag{11}$$

$$f'(0) = 0, \; f(0) = 0, \; y = 0, \tag{12}$$

$$f'(1) = U, \; f(1) = 1, \; y = 1, \tag{13}$$

where $U = -U^*/(v_s x)$ is the dimensionless velocity of the upper plate.

Using Eq. (11), a constraint on the pressure may be obtained

$$\frac{\partial^2 p}{\partial x\,\partial y} = 0, \tag{14}$$

and the written Eq. (10) may be written in the form

$$f''' + (f'^2 - ff'')\,\mathrm{Re} = k = \mathrm{const}. \tag{15}$$

The method of successive approximation is used to solve Eq.(15)with the n boundary conditions in Eqs. (12) and (13), under the assumption of small Reynolds number (Re << 1). Within the framework of the two approximations, it is found that

$$f = ay^2 - by^3 + \left[\frac{y^2}{10}\left(ab - \frac{2a^2}{3} - \frac{4b^2}{7}\right) + \right.$$
$$\left. + \frac{y^3}{6}\left(\frac{3a^2}{5} - \frac{4ab}{5} + \frac{3b^2}{7}\right) - \frac{a^2}{30}y^5 + \frac{ab}{30}y^6 - \frac{b^2}{70}y^7\right]\mathrm{Re} + \ldots, \tag{16}$$

$$k = -6b + \left(\frac{3a^2}{5} - \frac{4ab}{5} + \frac{3b^2}{7}\right)\mathrm{Re} + \ldots, \tag{16}$$

where $a = 3 - U$, $b = 2 - U$ are constants depending on the velocity of motion of the upper plate. When $U = 0$, Eq. (16) transforms to the solution of the corresponding problem for plane Poiseuille flow [2].

Taking account of Eq. (16), the solution of the mass-transfer equation is obtained from Eq. (9)

$$u = -x\left[2ay - 3by^2 + \left(\frac{1}{5}Ay + \frac{1}{2}By^2 - \frac{1}{6}a^2y^4 + \frac{1}{5}aby^5 - \frac{1}{10}b^2y^6\right)\mathrm{Re} + \ldots\right], \tag{17}$$

$$v = ay^2 - by^3 + \left(\frac{1}{10}Ay^2 + \frac{1}{6}By^3 - \frac{1}{30}a^2y^5 + \frac{1}{30}aby^6 - \frac{1}{70}b^2y^7\right)\mathrm{Re} + \ldots, \tag{18}$$

where $A = ab - \frac{2a^2}{3} - \frac{4b^2}{7}$; $B = \frac{3a^2}{5} - \frac{4ab}{5} + \frac{3b^2}{7}$.

The heat-conduction equation, Eq. (4), is now considered with the boundary conditions in Eqs. (7) and (8). The given problem is satisfied by a function T(y) of the form

$$T(y) = 1 - m\int_1^y \exp\left[\mathrm{Pe}\int_1^y v(y)\,dy\right]dy. \tag{19}$$

Substituting Eq. (18) into Eq. (19) and integrating, the temperature distribution over the height of the slit channel is obtained

$$T(y) = 1 - m\exp\mathrm{Pe}\left[\frac{b}{4} - \frac{a}{3} - \left(\frac{A}{30} + \frac{B}{24} - \frac{a^2}{180} + \frac{ab}{210} - \frac{b^2}{560}\right)\mathrm{Re}\right]\int_1^y \exp\mathrm{Pe}\left[\frac{ay^3}{3} - \frac{by^4}{4} + \left(\frac{Ay^3}{30} + \frac{By^4}{24} - \frac{a^2y^6}{180} + \frac{aby^7}{210} - \frac{b^2y^8}{560}\right)\mathrm{Re}\right]dy. \tag{20}$$

Now Eq. (20) is reduced to simpler form. The asymptotic solution of the mass-transfer equation is found under the assumption that the Reynolds number is small. If it is assumed that the inequality Re << 1 holds on account of the smallness of the product Hv_s (small slit height and slight sublimation velocity of the solidified gas), it is natural to assume that Pe << 1. In this case, the product RePe may be neglected, and Eq. (20) is significantly simplified

$$T(y) = 1 - m\exp\mathrm{Pe}\left(\frac{b}{4} - \frac{a}{3}\right)\int_1^y \exp\mathrm{Pe}\left(\frac{ay^3}{3} - \frac{by^4}{4}\right)dy. \tag{21}$$

If the dependence $\exp\alpha = 1 + \alpha$ is then used, where α is an infinitely small quantity, a formula that is simple and expedient for practical calculations is obtained

$$T(y) = 1 - m\left\{y - 1 + \mathrm{Pe}\left[\frac{a}{12}(y^4 - 1) - \frac{b}{20}(y^5 - 1) + \left(\frac{b}{4} - \frac{a}{3}\right)(y - 1)\right]\right\}. \tag{22}$$

Curves of T(y) plotted from Eq. (22) with different values of the upper-plate velocity are shown in Fig. 2, from which it is evident that the absence of sublimation (Pe = 0) leads to a linear distribution of the temperature over the slit height. The presence of evaporational cooling considerably reduces the temperature of the heated channel wall (y = 0). Motion of the subliming plate intensifies this process, giving the best results at high shift velocities.

NOTATION

$x = x^*/H$, $y = y^*/H$, dimensionless physical coordinates; H, slit height; $v = v^*/v_s$, $u = u^*/v_s$, dimensionless transverse and longitudinal velocity components; v_s, velocity of gas injection from the subliming wall; $p = p^*/\rho v_s$, dimensionless pressure; ρ, density; $m = \mathrm{Pe}r_s/T_sc_p$, dimensionless complex; $r_s = q/(v_s \rho)$, heat of sublimation; c_p, specific heat at constant pressure; T_s, temperature at the subliming surface; $T = T^*/T_s$, dimensionless temperature; $a = \lambda/(c_p \rho)$, thermal diffusivity; λ, thermal conductivity; ν, kinematic viscosity.

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DENSITY OF ADSORBED WATER IN DISPERSED SYSTEMS.

II. MONTMORILLONITE

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The density has been determined for water adsorbed on montmorillonite from the changes in density in the dispersion medium and in the lattice deformation in the solid phase.

The density distribution has been derived for hydrated montmorillonite (Fig. 1) for critical mass of the adsorbent (here 2 g) as measured with nitrobenzene, and the change in volume of the unit cell during adsorption has also been measured in water and nitrobenzene (given in [1]), which enables one to calculate the density d_w of the adsorbed water as follows:

$$d_w = \frac{0.01\,(P - P_0)\, d_0 d \dfrac{V_0}{V_i}}{0.01 P d_0 \dfrac{V_0}{V_i} + d_0 \dfrac{V_0}{V_i} - d}, \tag{1}$$

where P is the water content in %; P_0, amount of water corresponding to the first adsorbed molecules, where the density change in the dispersion medium has not yet occurred, and in our case $P_0 = 1\%$; d_0 and d, densities of the adsorbent measured at P_0 and P correspondingly; and V_i and V_0, unit-cell volumes measured at P and P_0 in the adsorption of water and organic compounds.

An essentially new situation is introduced with P_0 into the calculation of the adsorbed-water density. It should be noted that $P_0 = 0$ for some adsorbents. For example, this parameter was not introduced in [2] in estimating the density of water adsorbed on kaolinite, although in an earlier study [3] concerned with the density of water adsorbed on montmorillonite, the estimates were made without allowance for the lattice deformation, and the total mass of water was reduced by P_0 (it was there denoted by P_h), but no adequate basis for this was given.

In fact, the density of the adsorbed water in (1) is referred to the mass $(P - P_0)$, i.e., it assumed that all the water is divided into two separate components at any level of content; 1) the initial groups firmly bound molecules with mass P_0, which are not adsorption centers for the subsequent molecules, and 2) all subsequent molecules with mass $(P - P_0)$, which form a single ensemble in the sorption.

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